Cyclic cohesive model for fatigue crack growth in concrete

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DTU

- Concrete highways and bridges
 - app. 5,000,000-50,000,000 passes over critical point during design period
- Port- and industrial pavements
 - app. 250,000-5,000,000 passes over critical point during design period
- Airport pavements
 - app. 10,000-250,000 passes over critical point during design period



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Motivation

Concrete subjected arbitrary loading

..moreover, some concrete structures are subjected to a wide spectrum of loads and load configurations..



Motivation

Design methodology



- Mechanical part: Measured or calculated response (stress or strain level, stress intensity etc.)
- Empirical part: Transfer function (S-N curve) converting the response into number of allowable load cycles
- The M-E models does not distinguish between elastic and inelastic work
- Model parameters A, B and C are regression constants without physical meaning



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M-E models vs. full-scale experiments



- The M-E models have difficulties in accurately predicting realistic behaviour
- Fatigue and monotonic behaviour are badly linked

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Fracture process zone In order to create a modelling framework that accounts for the material behaviour during arbitrary loading one need to link:

- The development of the fracture process zone (FPZ);
- the damage of the existing fracture process zone (FPZ), i.e. reduction of the bridging stresses;
- and the monotonic material characteristics



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- (i) Fiber level: Develop a cyclic cohesive model considering a fiber of concrete material in uni-axial tension
- (ii) Hinge level: Incorporate the fiber response in a hinge model for description of bending fracture
- (iii) Beam level: For structural analysis, the hinge model is then applied as a constitutive model in a non-linear beam element



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(i) Fiber level: Energy based cyclic cohesive model



- Reduction of bridging stresses is controlled by deterioration of the fracture energy
- Damage initiation is given by simple relations from geometrical interpretation
- Unloading- and reloading follow straight lines
- Modifications of the model possible, e.g. adjusting the unloading response
- The deterioration of the fracture energy is controlled by the work during fatigue loading, defined as the area below the curve
- A simple linear relationship is selected, where k_{fat} is used to scale the energy dissipated during fatigue loading as a fraction of accumulated work



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Numerical study of cracked concrete in uni-axial tension (Plizzari et al., 1997)



- Good fit obtained with the secant unloading scheme (suitable for design) ۲
- ۲ Realistic response with unloading towards σ_k^u (suitable for inverse analysis)
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(ii) Hinge level: Hinge model for description of bending fracture



The hinge model - basic assumptions

• A crack influence the stress and strain field of a structure locally

- The discontinuity is expected to vanish outside a certain width, defined as the hinge width s
- Thus, the propagation of a crack can be modelled as a hinge, whereas the rest of the beam can be treated as elastic bulk material



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Hinge level



From the elongation of a fiber located at, y_i , and the softening response, ۲ $\sigma_i = b_i - a_i s,$

- the relevant stresses can be found for each fiber in the cracked phases ۲
- The sectional forces can be found from integration over the hinge height, i.e. the ۲ sum of contributions from each strip

$$N(\bar{\varepsilon}_{0},\bar{\kappa}) = t \int_{-h/2}^{h/2} \sigma_{c} \, \mathrm{d}y = \sum_{i=1}^{n} N_{i} \quad , \quad M(\bar{\varepsilon}_{0},\bar{\kappa}) = t \int_{-h/2}^{h/2} \sigma_{c}y \, \mathrm{d}y = \sum_{i=1}^{n} M_{i}$$



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• Considering a hinge subjected to constant curvature loading

- Visualising the response of a fiber on the upper part of the hinge (green) and the bottom fiber (red)
- The bottom fiber enters fatigue after the first load step, as the lower part of the hinge deteriorate, new fibers in the upper part are activated
- The upper fiber is first in compression, then in linear elastic tension, before entering a short stage of low-cyclic damage and finally fatigue loading the model accounts for all the cracked phases in a unified manner



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(iii) Beam level: Hinge as constitutive model in a non-linear beam element



- The proposed hinge is implemented in a user built finite element code applying a three node beam element
- The underlying description of the hinge is based on the formation of discrete cracks
- However, when implemented into a beam element, the constitutive behaviour of the hinge is smeared (smooth)
- Making the model applicable for structural analysis, e.g. area outside the loaded points (where the moment distribution is no longer constant)



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Experiments

- $\bullet\,$ Damage parameter k_{fat} calibrated to match the first experiment and then kept constant
- The model is able to capture the crack growth development and number of cycles to failure
- The initial crack length increase with increasing fatigue load
- The crack growth rate increase with increasing fatigue load
- The fatigue life increase for decreasing fatigue load





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- FE hinge $0.97P_u$
- FE hinge $0.93P_u$

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Summary

Conclusions



- (i) A general cyclic cohesive model was developed based on energy considerations
- (ii) The cyclic fiber response was incorporated in a hinge model for description of bending fracture
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- (i) The selected format is general and consistent and accounts for the material behaviour in all the cracked phases
- (ii) The proposed model shows satisfactory results when compared to experiments with cracked plain concrete cylinders in uni-axial tension
- (iii) However, experimental validation of the fatigue damage parameter k_{fat} is difficult due to lack of sufficient data
- (iv) Main characteristic features, such as initial crack length and fatigue crack growth rate, can be simulated with the finite element hinge model
- (v) The results obtained are encouraging and show that the methodology is well suited for practical use

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