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On Fracture Testing of Sandwich Face/Core Interface using the DCB-UBM Methodology in Fatigue

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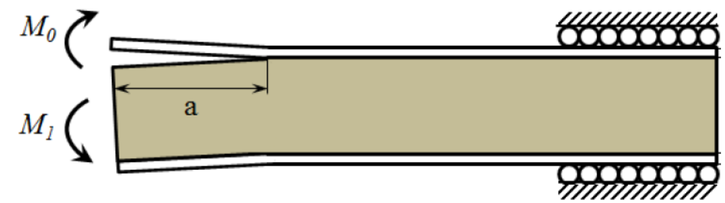
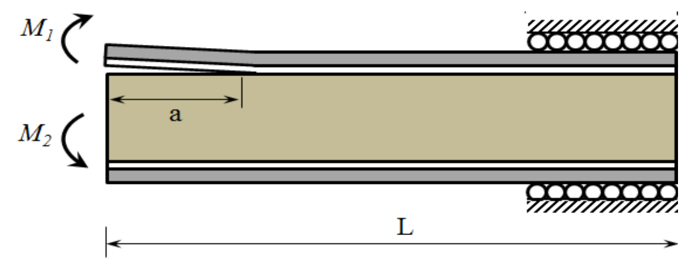
Background and Motivation

- Sandwich composites applications; widely in wind turbine, aerospace and marine industry and growing
- Increasingly optimized structures to yield minimum weight and maximum performance
- Emphasizes the need for adequate *fracture mechanical tools* for damage assessment
  - In particular to assess *debond induced* damages
- Measurements of **fracture properties** are therefore an increasingly important task
  - *Fracture toughness*
  - *da/dN diagrams*

Sandwich Double Cantilever Beam with Uneven Bending Moments (DCB-UBM) specimen

- Pure moments applied at the crack flanks
- No transverse forces
- G-controlled by nature – no need for crack length measurements
- Stable crack growth
  - Constant mode-mixity enabled by fixing the ratio of moments, **MR** = M_1/M_2
- Analytical foundation
  - Closed-form solutions for ERR and mode-mixity phase angle





Saseendran, V., Berggreen, C., and Carlsson, L. A., "Fracture Mechanics Analysis of Reinforced DCB Sandwich Debond Specimen Loaded by Moments" *AIAA Journal*, 2017

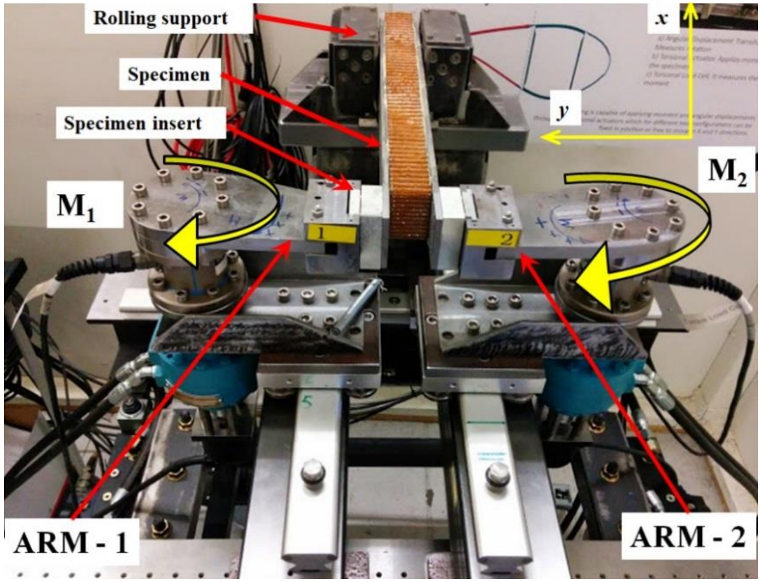
$$G = P^2\left(\frac{L_1}{(\bar{E}h)_d^2} + \frac{V_1}{(\bar{E}h)_s^2} + \frac{V_2\Delta_1^2}{H_s^2} + \frac{V_3\Delta_1}{(\bar{E}h)_s H_s}\right) + M_d^2\left(\frac{L_2}{H_d^2} + \frac{V_2}{H_s^2}\right) + M_d P\left(\frac{2V_2\Delta_1}{H_s^2} + \frac{L_3}{(\bar{E}h)_d H_d} + \frac{V_3}{(\bar{E}h)_s H_s}\right)$$

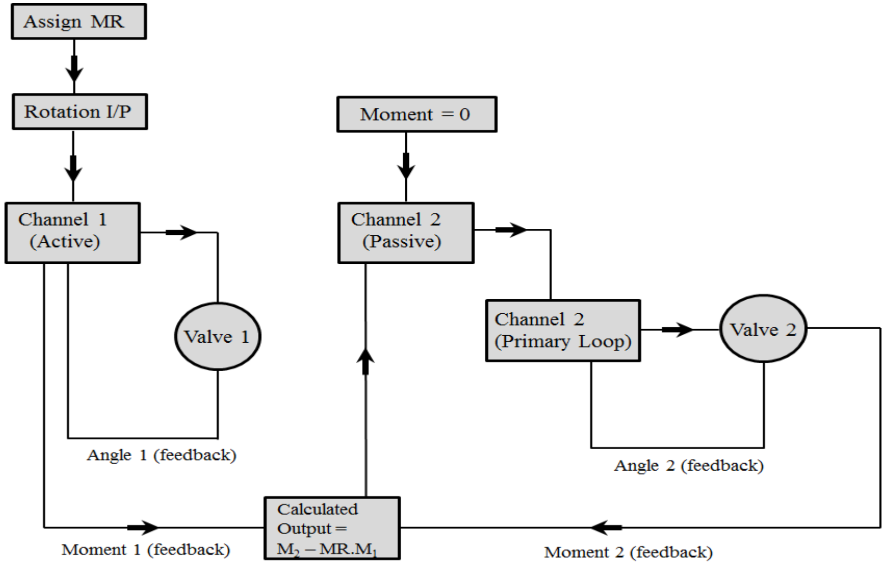
$$\psi = \tan^{-1}\left[\frac{\lambda \sin\omega - \cos(\omega+\gamma)}{\lambda\cos\omega + \sin(\omega+\gamma)}\right]$$

$$\lambda = -\frac{P^*}{M_d^*}\sqrt{\frac{a_1}{a_2}} \qquad \sin\gamma = \frac{-a_3}{2\sqrt{a_1 a_2}}$$

DCB-UBM: Fatigue Testing Algorithm

- Moments applied using independent torsional actuators
- Tests carried out in Rotation control
- Rotation I/p provided to Arm 1
  - Arm 2 follows Arm 1 maintaining constant MR throughout the test
- Two independent channels work in tandem to achieve constant MR.
  - Cascade control
  - Cannot control both arms in moment mode simultaneously.





DCB-UBM: Fatigue Testing Algorithm

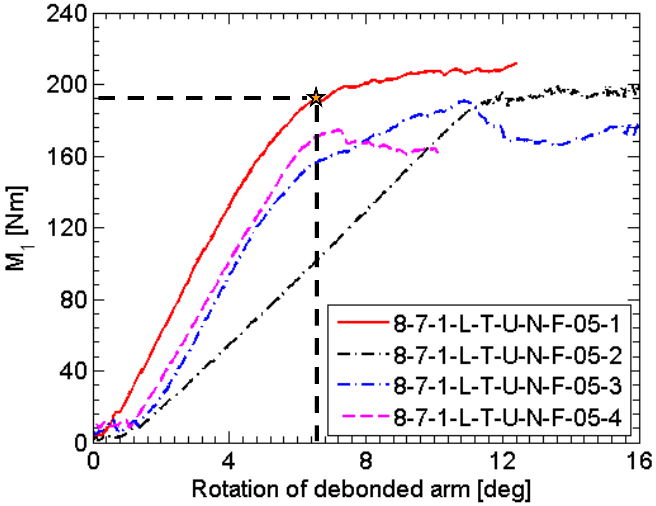
- The relationship b/w G vs M exploited

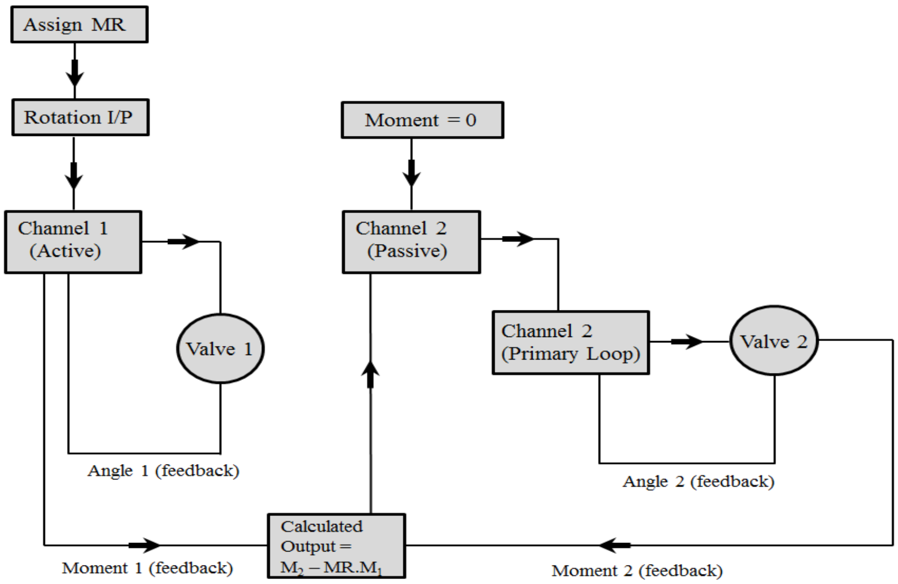
$$G = \frac{M_1^2}{2}\frac{dC_1}{dA} + \frac{M_2^2}{2}\frac{dC_2}{dA} + \frac{M_3^2}{2}\frac{dC_3}{dA} = \frac{1}{2b}\left[\frac{M_1^2}{(EI)_1} + \frac{M_2^2}{(EI)_2} + \frac{M_3^2}{(EI)_3}\right]$$

  - Critical moment associated with crack initiation identified using a static test (M_{1c})

$$G_c = \frac{1}{2b}\left[\frac{M_{1c}^2}{(EI)_1} + \frac{M_{2c}^2}{(EI)_2} + \frac{M_{3c}^2}{(EI)_3}\right]$$

  - Now, say for 50% of *Gc* identify moment corresponding to 70% ($M_{1\_50\%}$)
  - Therefore, angle (A_1) which achieves $M_{1\_50\%}$ for a certain crack length is identified
  - Sine curve at 1 Hz executed with continuous crack monitoring!

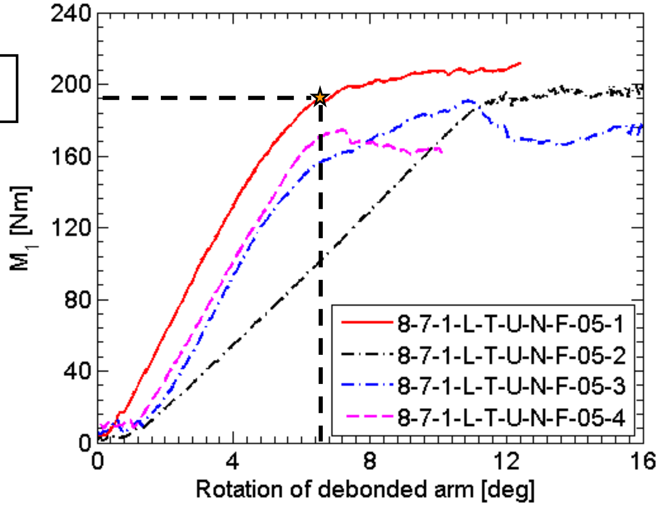




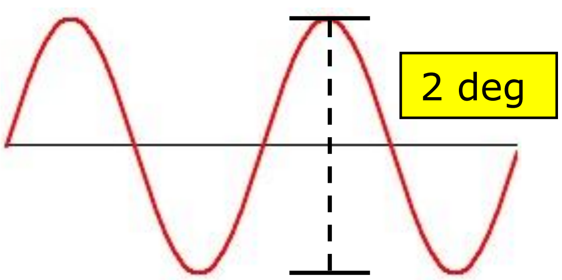
Mixed-mode screening: Pilot Fatigue Testing

- Pilot testing carried out on round robin specimen
  - Static data already available!
  - HRH-10-3.2-48 (3.2 mm cell size, 48 kg/m^3 density) core
  - h_f = 0.79 mm, h_c = 25.4 mm, E_c = 138 MPa

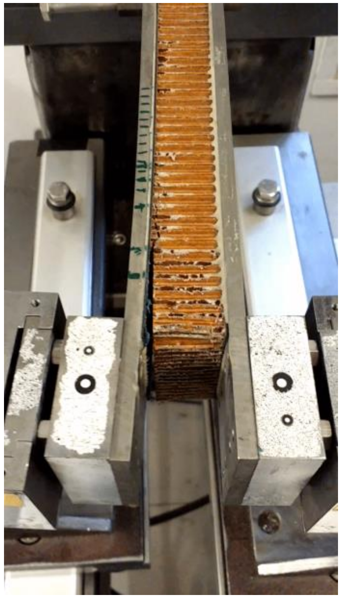
G_{Ic} ~ 1000 J/m^2



- Angle i/p varied to obtain various ΔG values and crack increment monitored.
  - MR kept constant (hence mode-mixity)
  - Sinusoidal cycle with angle diff. = 2 deg.
  - Reduce inertia effects by minimizing arm rotation
  - Incremental crack positions are pre-marked on the specimen



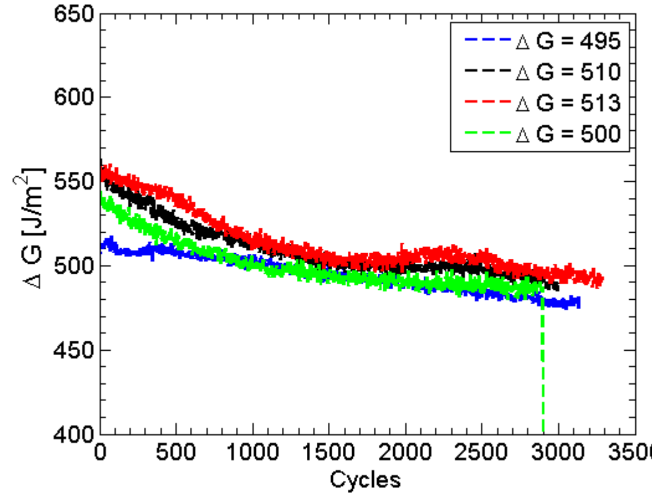
$$G_c = \frac{1}{2b}\left[\frac{M_{1c}^2}{(EI)_1} + \frac{M_{2c}^2}{(EI)_2} + \frac{M_{3c}^2}{(EI)_3}\right]$$



Mixed-mode screening: Pilot Fatigue Testing

- Interface crack propagation

ΔG (mean) (J/m²)	Δa (mm)
495	10.10
510	12.10
513	15.60
500	10.60



- However, angle not updated when ΔG drops →
  - Algorithm can be updated to increase the angle to keep constant ΔG (crude form!)
  - Wagon displacements are minimal at lower rotation (small friction component)

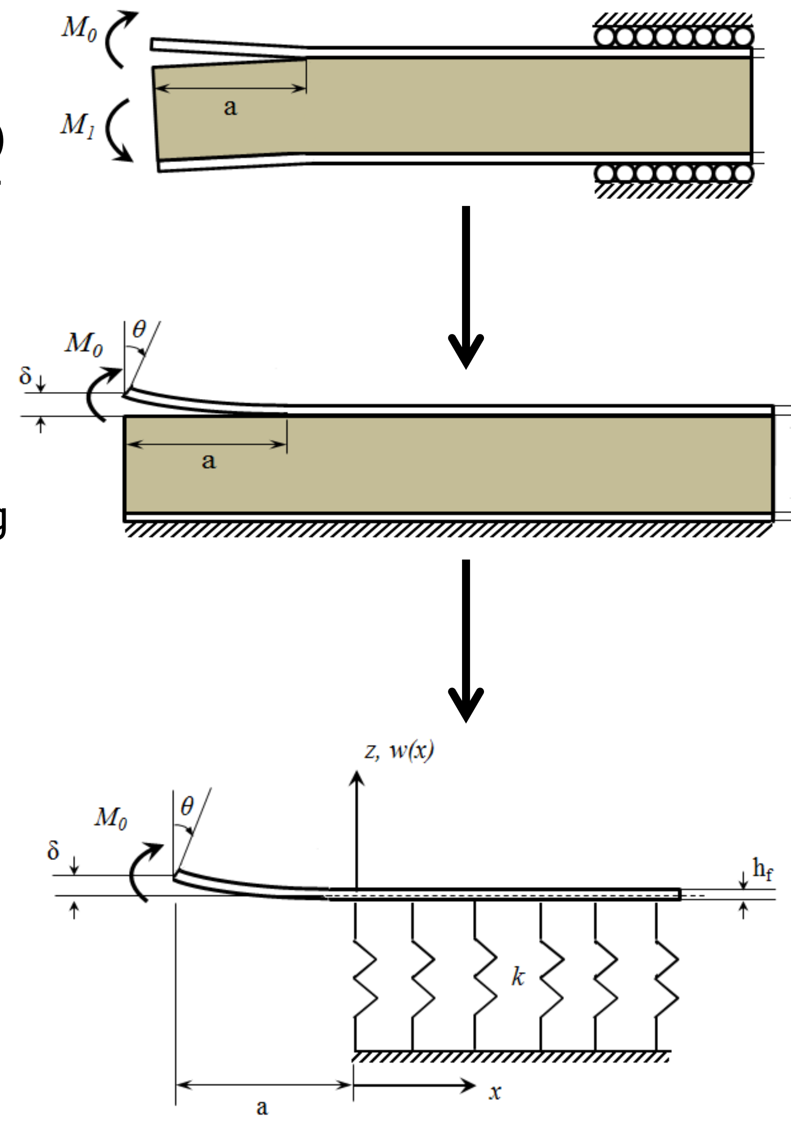
- Program needs to be updated when ΔG drops based on CTOD (***on-going***)
  - Derivation of full kinematic model (***on-going***)

DCB-UBM: Moment Loaded SCB

- DCB loaded with Un-even Bending Moments (DCB-UBM) split into two parts:
  - Upper beam (resting on elastic foundation)
  - Lower beam (comprising of core and lower face sheet)

- For analysis the upper beam (face sheet) is considered to be resting on an elastic foundation
  - Winkler foundation model can be utilized
  - The Winkler model is solved by considering a semi-infinite elastic foundation
  - Governing differential equation consisting of two parts: debonded (*-a < x < 0*) and elastic foundation (*0 < x < ∞*):

$$EI\frac{d^4w}{dx^4} + kH(x)w = 0 \qquad H(x) = \begin{cases} 1, x > 0 \\ 0, x < 0 \end{cases}$$



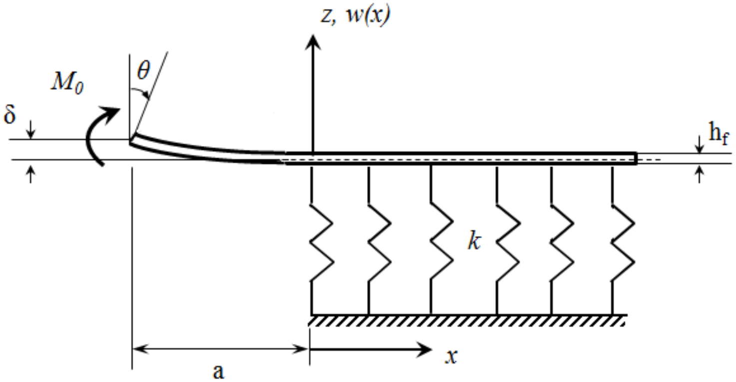
Moment Loaded SCB: Solution to Winkler mechanical model

- Governing differential equation:

$$EI\frac{d^4w}{dx^4}+kH(x)w=0 \qquad H(x)=\begin{cases}1, x>0\\0, x<0\end{cases}$$

- General solution:

$$w(x)=B_1e^{\lambda x}\cos(\lambda x)+B_2e^{\lambda x}\sin(\lambda x)+B_3e^{-\lambda x}\cos(\lambda x)+B_4e^{-\lambda x}\sin(\lambda x)$$



- For semi-infinite beam, end effects are neglected and exponentially decaying terms are only retained:

$$w(x)=B_3e^{-\lambda x}\cos(\lambda x)+B_4e^{-\lambda x}\sin(\lambda x)$$

$$\lambda=\left[\frac{3k}{E_f t_f^3 b}\right]^{\frac{1}{4}}=\left[\frac{3E_c}{t_c t_f^3 E_F}\right]^{\frac{1}{4}}$$

- Progressive differentiation yields:

$$\theta(x)=\frac{dw(x)}{dx}=-B_3\lambda f_3(\lambda x)+B_4\lambda f_4(\lambda x)$$

$$M(x)=-EI\frac{dw^2(x)}{dx^2}=-\frac{B_3k}{2\lambda^2}f_2(\lambda x)+\frac{B_4k}{2\lambda^2}f_1(\lambda x)$$

$$V(x)=\frac{dM(x)}{dx}=-\frac{B_3k}{2\lambda}f_4(\lambda x)+\frac{B_4k}{2\lambda}f_3(\lambda x)$$

$$f_1(\lambda x)=e^{-\lambda x}\cos(\lambda x)$$
$$f_2(\lambda x)=e^{-\lambda x}\sin(\lambda x)$$
$$f_3(\lambda x)=e^{-\lambda x}\left(\cos(\lambda x)+\sin(\lambda x)\right)$$
$$f_4(\lambda x)=e^{-\lambda x}\left(\cos(\lambda x)-\sin(\lambda x)\right)$$

- Solve for B_3 and B_4 by BCs: $V = 0$ and $M = M_0$ at $x = 0$.

Moment Loaded SCB: Solution to Winkler mechanical model

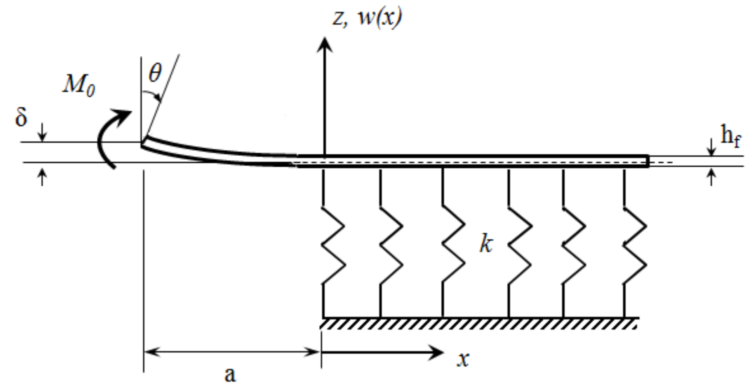
- Solving for constants B_3 and B_4 yields deflection, rotation, moment and shear in the foundation part:

$$\mathrm{w}(x)=\frac{M_o 2\lambda^2}{k}\left(f_1(\lambda\,\mathrm{x})-f_2(\lambda\,\mathrm{x})\right)\qquad (0 \le x \le \infty)$$

$$\theta(x)=\frac{dw(x)}{dx}=-\frac{M_o 2\lambda^3}{k}\left(f_3(\lambda\,\mathrm{x})-f_4(\lambda\,\mathrm{x})\right)$$

$$M(x)=EI\,\frac{dw^2(x)}{dx^2}=-M_o\left(f_2(\lambda\,\mathrm{x})-f_1(\lambda\,\mathrm{x})\right)$$

$$V(x)=EI\,\frac{dw^3(x)}{dx^3}=-M_o\lambda\left(f_4(\lambda\,\mathrm{x})+f_3(\lambda\,\mathrm{x})\right)$$



$$f_1(\lambda\,\mathrm{x})=e^{-\lambda x}\cos(\lambda\,\mathrm{x})$$
$$f_2(\lambda\,\mathrm{x})=e^{-\lambda x}\sin(\lambda\,\mathrm{x})$$
$$f_3(\lambda\,\mathrm{x})=e^{-\lambda x}\left(\cos(\lambda\,\mathrm{x})+\sin(\lambda\,\mathrm{x})\right)$$
$$f_4(\lambda\,\mathrm{x})=e^{-\lambda x}\left(\cos(\lambda\,\mathrm{x})-\sin(\lambda\,\mathrm{x})\right)$$

- Deflection for the debonded part obtained by solving homogenous equation:

$$EI\,\frac{d^4w}{dx^4}=0$$

- General solution is of the form: $$w(x)=C_1\frac{x^3}{6}+C_2\frac{x^2}{2}+C_3x+C_4$$

- Constants C_1 and C_2 can be obtained from BCs: $V(x=0) = 0$ and $M(x = 0) = M_o$

Moment Loaded SCB: Solution to Winkler mechanical model

- Solving for constants C_1 and C_2 in:

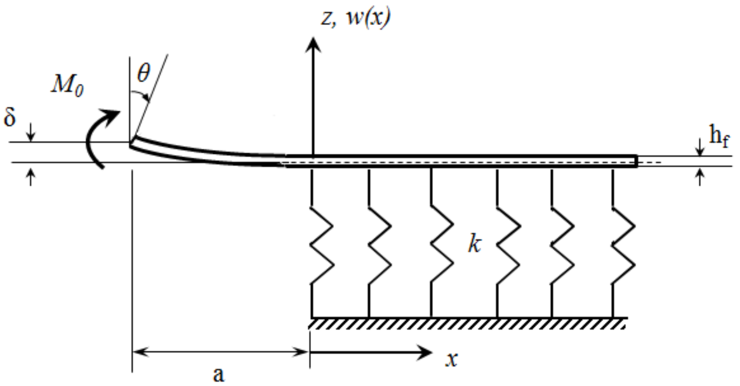
$$w(x) = C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4$$

- Must also ensure continuity with the foundation part, thus:
  - Deflection and the progressive derivatives must be continuous in the two intervals (-*a, 0*) and (*0, ∞*).

$$\mathrm{w}(x) = M_o\left[\frac{x^2}{2EI} - \frac{4\lambda^3 x}{k} + \frac{2\lambda^2}{k}\right] \qquad -a \le x \le 0$$

- The total deflection of a moment loaded SCB specimen is:

$$w(x) = M_o\begin{cases}\dfrac{x^2}{2EI} - \dfrac{4\lambda^3 x}{k} + \dfrac{2\lambda^2}{k} & (-a \le x \le 0)\\[2ex] \dfrac{2\lambda^2}{k}\left[f_1(\lambda x) - f_2(\lambda x)\right] & (0 \le x \le \infty)\end{cases}$$



Deflection and rotation of beam with a built-in end at x = 0, can be recovered from k -->∞

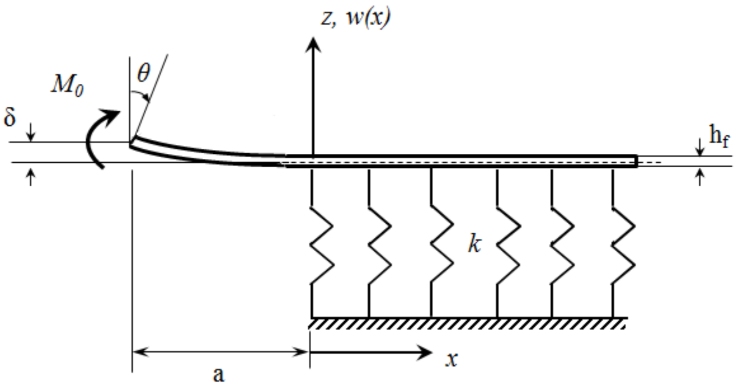
$$w(-\mathrm{a}) = M_0 a^2 / 2EI$$

$$\theta(-\mathrm{a}) = -M_0 / 2EI$$

Moment Loaded SCB: Compliance and Energy-release rate

- Compliance defined as rotation/moment:

$$C = \frac{|\theta(-a)|}{M_0}$$



- Rotation can be obtained from the deflection solved for Winkler model:

$$\theta(x) = \frac{dw}{dx} = M_o \begin{cases} \frac{x}{EI} - \frac{4\lambda^3}{k} & (-a \le x \le 0) \\ \frac{2\lambda^2}{k}\left[-f_3(\lambda x) - f_4(\lambda x)\right] & (0 \le x \le \infty) \end{cases}$$

- Hence at x = -a, the compliance is: $C = \frac{a}{EI} + \frac{4\lambda^3}{k}$

- Energy-release rate expressed as: $G = \frac{M^2}{2b}\frac{dC}{da} \longrightarrow G = \frac{M_o^2}{2bEI}$

Moment Loaded SCB: FE Analysis

- FE Analysis (2D) carried out with a *Al/H100 sandwich specimen*; h_f = 6.35 mm, h_c = 25.4 mm), M_o = 1 N mm/mm, final crack length = 50.8 mm (2 inch)

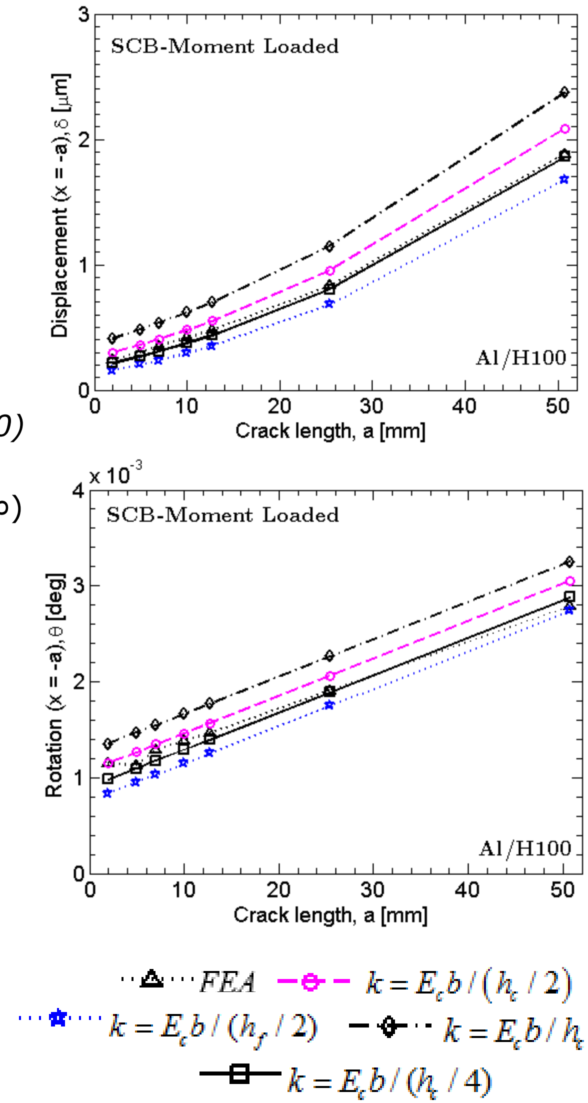
- Displacement and rotation obtained using algebraic expression obtained earlier at $x = -a$:

$$w(x) = M_o \begin{cases} \dfrac{x^2}{2EI} - \dfrac{4\lambda^3 x}{k} + \dfrac{2\lambda^2}{k} \\ \dfrac{2\lambda^2}{k}\left[f_1(\lambda x) - f_2(\lambda x) \right] \end{cases} \qquad \theta(x) = \frac{dw}{dx} = M_o \begin{cases} \dfrac{x}{EI} - \dfrac{4\lambda^3}{k} & (-a \le x \le 0) \\ \dfrac{2\lambda^2}{k}\left[-f_3(\lambda x) - f_4(\lambda x) \right] & (0 \le x \le \infty) \end{cases}$$

  - Both displacement and rotation increase w/t increasing crack lengths

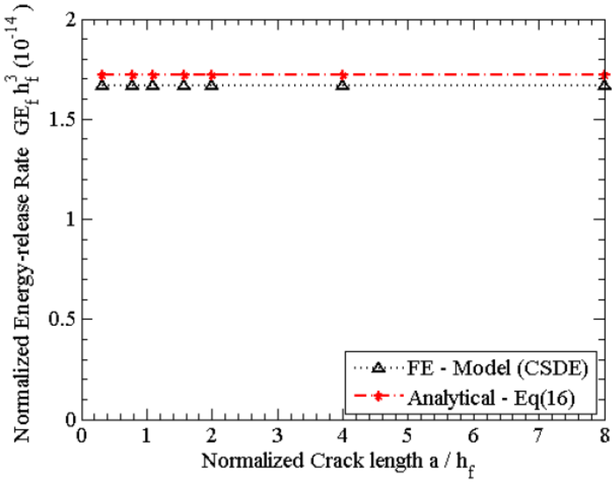
- The foundation modulus expression proposed gives good agreement with FE results!

$$k = \frac{E_c b}{h_c / 4}$$



Moment Loaded SCB: FE Analysis

- Energy-release rate expressed as: $G = \dfrac{M_o^2}{2bEI}$
- Independent of :
  - Crack length, *a*
  - Elastic foundation modulus, *k*
- *G* determined from FEA unaffected by crack length
  - *G* normalized with $E_f h_f^3$
- A difference in 3% is observed b/w FEA and analytical expression for all range of crack lengths



Conclusions and Future Work

- Pilot fatigue testing performed
  - Algorithm implemented using angle control
  - Way forward: maintain constant ΔG by updating angle or control using CTOD

- Winkler foundation model applied to a moment loaded SCB specimen was solved and compared with FEA
  - Current analysis is an initial step toward solving DCB-UBM
  - Analytical expressions compare fairly well with FE results

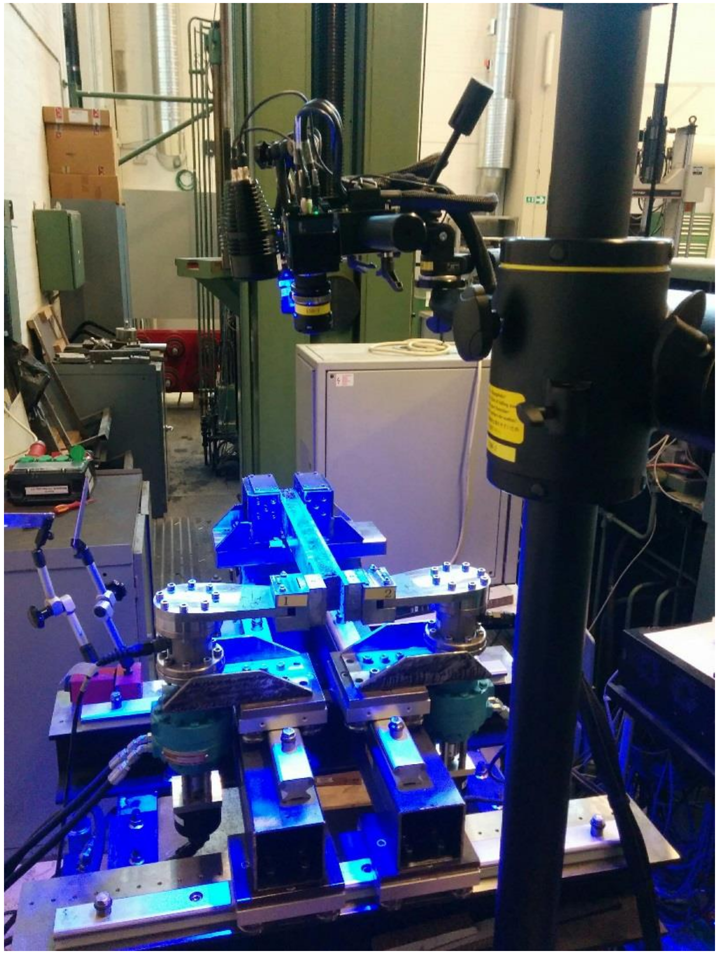
Future Work

- Derivation of DCB-UBM kinematics solving for the lower beam (on-going)
- Extension into fatigue, using kinematics to control on end openings (on-going)
- Fracture characterization at high/low temperatures (design of climatic chamber on-going)

THANK YOU FOR YOUR ATTENTION!

ACKNOWLEDGMENT

This work is supported by DCCSM. Support of VILLUM Ffoundation is also greatly acknowledged.



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Appendix

Sandwich DCB-UBM specimen reinforced with steel doublers

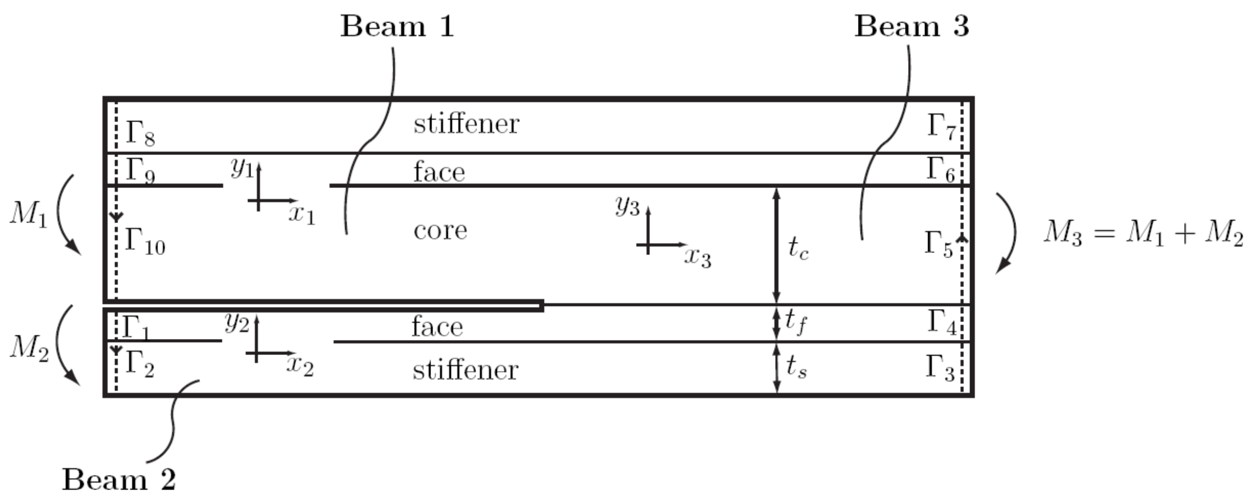
- Avoid yielding in reinforcements and excessive rotations
- Possible to account for *thin face sheets*
- Energy Release Rate (ERR) via J-integral calculation: *(Lundsgaard et al, 2007)*

$$J = \sum_{p=1}^{10} \frac{E_p {M_b}^2}{6(A_b D_b - {B_b}^2)^2} \left[{A_b}^2\left({y_{p-1}}^3 - {y_p}^3\right) - 3A_b B_b\left({y_{p-1}}^2 - {y_p}^2\right) + 3{B_b}^2\left(y_{p-1} - y_p\right)\right]$$

$$A_b = \sum_{k=1}^{n} \bar{E}_k (y_k - y_{k-1})$$

$$B_b = \frac{1}{2}\sum_{k=1}^{n} \bar{E}_k ({y_k}^2 - {y_{k-1}}^2)$$

$$D_b = \frac{1}{3}\sum_{k=1}^{n} \bar{E}_k ({y_k}^3 - {y_{k-1}}^3)$$



Sandwich DCB-UBM specimen
Novel compact fatigue rated rig

Specifications:

- Low friction roller wagon/rail system
  - Two torsional actuators (700 Nm)
  - Two 10 [L/min] servo-valves
  - Two 565 [Nm] torsional load cells
- Bi-axial servo-hydraulic controller (MTS FlexTest 40)
- Conditional control (CASCADE)
  - Rotation controlled tests